1. A curve C has parametric equations

$$x = \frac{t^2 + 5}{t^2 + 1} \qquad y = \frac{4t}{t^2 + 1} \qquad t \in \mathbb{R}$$

Show that all points on C satisfy

$$(x-3)^2 + y^2 = 4$$

(3)

Rearrange for t^2:

$$x = \frac{t^2+5}{t^2+1}$$

$$x(t^2+1) = t^2+5$$

$$xt^2 + x = t^2 + 5$$

$$xt^2 - t^2 = 5 - x$$

$$t^2(x-1) = 5-x$$

$$t^2 = \frac{5-x}{x-1}$$

$$y = \frac{4t}{t^2+1}$$

$$y^2 = \frac{16t^2}{(t^2+1)^2}$$

eliminate t

$$y^2 = \frac{16\left(\frac{5-x}{x-1}\right)}{\left(\frac{5-x}{x-1}+1\right)^2} \quad 1$$

$$\frac{5-x}{x-1} + 1 = \frac{5-x}{x-1} + \frac{x-1}{x-1} = \frac{4}{x-1}$$

$$y^2 = \frac{16(5-x)}{x-1} \times \left[\frac{(x-1)}{4}\right]^2$$

$$= (5-x)(x-1) \quad 1$$

$$= 5x - 5 - x^2 + x$$

$$y^2 = -x^2 + 6x - 5$$

$$y^2 = -x^2 + 6x - 5$$

$$y^2 + x^2 - 6x + 5 = 0$$

complete square

$$y^2 + (x-3)^2 - 9 + 5 = 0$$

$$y^2 + (x-3)^2 = 4 \quad 1$$

as required

2. The curve C has parametric equations

$$x = t^2 + 6t - 16 \qquad y = 6\ln(t+3) \qquad t > -3$$

(a) Show that a Cartesian equation for C is

$$y = A\ln(x+B) \qquad x > -B$$

where A and B are integers to be found.

(3)

The curve C cuts the y-axis at the point P

(b) Show that the equation of the tangent to C at P can be written in the form

$$ax + by = c\ln 5$$

where a, b and c are integers to be found.

(4)

a) Method: rearrange x to find $t+3$, then substitute into y.

$$x = t^2 + 6t - 16$$
$$x = (t+3)^2 - 9 - 16$$
$$x = (t+3)^2 - 25 \quad ①$$
$$(t+3)^2 = x + 25$$
$$(t+3) = (x+25)^{1/2} \quad ①$$

$$y = 6\ln(t+3) = 6\ln(x+25)^{1/2}$$

$$y = 3\ln(x+25) \quad ①$$

b) when $x = 0$, $y = 3\ln 25 = 6\ln 5$ ① $\quad 3\ln 25 = 3\ln 5^2 = 6\ln 5$

$$\frac{dy}{dx} = \frac{3}{x+25} = \frac{3}{0+25} \quad ①$$

$$y - 6\ln 5 = \frac{3}{25}(x - 0) \quad ①$$

$$25y - 150\ln 5 = 3x$$

$$25y - 3x = 150\ln 5 \quad ①$$